Playing with fermion couplings in Higgsless models

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We discuss the fermion couplings in a four dimensional $SU(2)$ linear moose model by allowing for direct couplings between the left-handed fermions on the boundary and the gauge fields in the internal sites. This is realized by means of a product of non linear σ-model scalar fields which, in the continuum limit, is equivalent to a Wilson line. The effect of these new non local couplings is a contribution to the ϵ_3 parameter which can be of opposite sign with respect to the one coming from the gauge fields along the string. Therefore, with some fine tuning, it is possible to satisfy the constraints from the electroweak data.

I. INTRODUCTION

Higgsless models [1, 2, 3, 4][5, 6, 7, 8, 9] have been recently considered as an alternative to the standard electroweak symmetry breaking mechanism. The corresponding effective theories are strongly interacting and share some similarities with the previously proposed technicolor models. Higgsless models are formulated as gauge theories in a five dimensional space and, after decompactification, describe a tower of Kaluza Klein (KK) excitations of the standard electroweak gauge bosons. These theories can also be understood as four dimensional deconstructed [10, 11, 12, 13] [14, 15, 16, 17, 18, 19, 20, 21] theories in the context of linear moose models. One of the interesting features of the Higgsless models is the possibility to delay the unitarity violation scale via the exchange of massive KK modes [1, 22, 23, 24, 25, 26, 27, 28]. However, in the simplest version of these models, it is difficult to reconcile a delayed unitarity with the electroweak constraints: in fact the ϵ_3 parameter tends to get a large contribution. For instance in the framework of models with only ordinary fermions it is possible to get small or zero ϵ_3 [17], at the expenses of having a unitarity bound as in the Standard Model (SM) without the Higgs, that is of the order of 1 TeV. A recent solution to the ϵ_3 problem which does not spoil the unitarity requirement at low scales, has been found by delocalizing the fermions in five dimensional

theories [29, 30]. In this paper we consider a linear moose model and we try to obtain a solution by introducing direct couplings (allowed by the symmetry of the model) between ordinary left-handed fermions and the gauge vector bosons along the moose string. This is possible by defining these couplings in terms of a product of non linear σ-model scalar fields which, in the continuum limit becomes a Wilson line. These interactions have been previously considered within a simple version of the moose models, the so-called BESS model [31, 32]. Since the contribution to the ϵ_3 parameter coming from fermions can be of opposite sign with respect to the one coming from the heavy vector mesons, typical of the moose, in principle there can be cancellations, though at the expenses of some fine tuning. This implies that the masses of the heavy vector mesons can be kept sufficiently low such that one can raise up the unitarity limit. In our solution the fermions live in four dimensions and no fermion KK excitations are present.

After reviewing the linear moose framework in Section II we introduce the new couplings of the left-handed fermions to the gauge bosons in Section III. In Section IV we study the low energy limit of the model and we derive the corresponding effective lagrangian containing only the SM fields. The kinetic terms of the effective lagrangians are not in the canonical form, therefore in Section V we proceed to a finite renormalization of the fields. In Section VI we calculate the ϵ parameters in terms of the coupling constants appearing in the original model. In Section VII we study some particular models according to the variation of the couplings along the string, and we show that there is in fact some space for cancellation, satisfying the experimental bounds. In Section VIII we study the continuum limit. Conclusions are given in Section IX. In Appendix A we decouple the heavy particles by finding explicit expressions for the corresponding fields in terms of the SM gauge fields. Finally in Appendix B we derive the form of the low-energy four-fermion interaction coming from the direct couplings of the fermions to the gauge bosons. This term provides a contribution to the definition of the Fermi constant.

II. REVIEW OF THE LINEAR MOOSE MODEL FOR THE ELECTROWEAK SYMMETRY BREAKING

Let us briefly review the linear moose model based on the $SU(2)$ symmetry. Following the idea of dimensional deconstruction [10, 11, 12, 13], the hidden gauge symmetry approach

applied to the strong interactions [14, 16, 33, 34, 35] and to the electroweak symmetry breaking [16, 31, 36], we consider $K+1$ non linear σ-model scalar fields Σ_i, $i = 1, \cdots, K+1$, K gauge groups, G_i, $i = 1, \cdots, K$ and a global symmetry $G_L \otimes G_R$. A minimal model of electroweak symmetry breaking is obtained by choosing $G_i = SU(2)$, $G_L \otimes G_R = SU(2)_L \otimes SU(2)_R$. The SM gauge group $SU(2)_L \times U(1)_Y$ is obtained by gauging a subgroup of $G_L \otimes G_R$. The Σ_i fields can be parameterized as $\Sigma_i = \exp(i/(2f_i)\vec{\pi}_i \cdot \vec{\tau})$ where $\vec{\tau}$ are the Pauli matrices and f_i are $K+1$ constants that we will call link couplings.

The transformation properties of the Σ_i fields are the following

$$\Sigma_1 \rightarrow L\Sigma_1 U_1^\dagger,$$
$$\Sigma_i \rightarrow U_{i-1}\Sigma_i U_i^\dagger, \quad i = 2, \cdots, K,$$
$$\Sigma_{K+1} \rightarrow U_K \Sigma_{K+1} R^\dagger, \tag{1}$$

with $U_i \in G_i$, $i = 1, \cdots, K$, $L \in G_L$, $R \in G_R$.

The lagrangian of the linear moose model for the gauge fields is given by

$$\mathcal{L} = \sum_{i=1}^{K+1} f_i^2 \text{Tr}[D_\mu \Sigma_i^\dagger D^\mu \Sigma_i] - \frac{1}{2}\sum_{i=1}^{K}\text{Tr}[(F_{\mu\nu}^i)^2] - \frac{1}{2}\text{Tr}[(F_{\mu\nu}(\tilde{W}))^2] - \frac{1}{2}\text{Tr}[(F_{\mu\nu}(\tilde{Y}))^2], \tag{2}$$

with the covariant derivatives defined as follows

$$D_\mu \Sigma_1 = \partial_\mu \Sigma_1 - i\tilde{g}\tilde{W}_\mu \Sigma_1 + i\Sigma_1 g_1 V_\mu^1,$$
$$D_\mu \Sigma_i = \partial_\mu \Sigma_i - ig_{i-1}V_\mu^{i-1}\Sigma_i + i\Sigma_i g_i V_\mu^i, \quad i = 2, \cdots, K,$$
$$D_\mu \Sigma_{K+1} = \partial_\mu \Sigma_{K+1} - ig_K V_\mu^K \Sigma_{K+1} + i\tilde{g}'\Sigma_{K+1}\tilde{Y}_\mu, \tag{3}$$

where $V_\mu^i = V_\mu^{ia}\tau^a/2$ and g_i are the gauge fields and gauge coupling constants associated to the groups G_i, $i = 1, \cdots, K$, and $\tilde{W}_\mu = \tilde{W}_\mu^a \tau^a/2$, $\tilde{Y}_\mu = \tilde{\mathcal{Y}}_\mu \tau^3/2$ are the gauge fields associated to $SU(2)_L$ and $U(1)_Y$ respectively.

The model described by the lagrangian given in eq.(2) is represented in Fig. 1. Notice that the field defined as

$$U = \Sigma_1 \Sigma_2 \cdots \Sigma_{K+1} \tag{4}$$

is the usual chiral field: in fact it transforms as $U \rightarrow LUR^\dagger$ and it is invariant under the G_i transformations.

The mass matrix of the gauge fields can be obtained by choosing $\Sigma_i = I$ in eq.(2). We find

$$\mathcal{L}_{\text{mass}} = \sum_{i=1}^{K+1} f_i^2 \text{Tr}[(g_{i-1}V_\mu^{i-1} - g_i V_\mu^i)^2] \equiv \frac{1}{2}\sum_{i,j=0}^{K+1}(M_2)_{ij}V_\mu^i V^{\mu j}, \tag{5}$$

where we have defined $V^0_\mu = \tilde{W}_\mu$, $V^{K+1}_\mu = \tilde{Y}_\mu$, $g_0 = \tilde{g}$, $g_{K+1} = \tilde{g}'$, $f_0 = f_{K+2} = 0$ and

$$(M_2)_{ij} = g_i^2(f_i^2 + f_{i+1}^2)\delta_{i,j} - g_i g_{i+1} f_{i+1}^2 \delta_{i,j-1} - g_j g_{j+1} f_{j+1}^2 \delta_{i,j+1}. \tag{6}$$

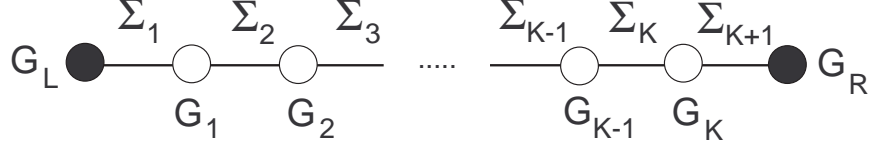
FIG. 1: *The linear moose model.*

III. COUPLINGS TO FERMIONS

In the following we will consider standard model fermions, that is: left-handed fermions ψ_L as $SU(2)_L$ doublets and singlet right-handed fermions ψ_R. The standard couplings are given by

$$\mathcal{L}_{fermions} = \bar{\psi}_L i\gamma^\mu(\partial_\mu + i\tilde{g}\tilde{W}_\mu + \frac{i}{2}\tilde{g}'(B-L)\tilde{\mathcal{Y}}_\mu)\psi_L + \bar{\psi}_R i\gamma^\mu(\partial_\mu + i\tilde{g}'\tilde{Y}_\mu + \frac{i}{2}\tilde{g}'(B-L)\tilde{\mathcal{Y}}_\mu)\psi_R \tag{7}$$

where B(L) is the baryon (lepton) number. These are the only fermions introduced in this model and they are coupled to the SM gauge fields through the groups $SU(2)_L$ and $U(1)_Y$ at the ends of the chain.

We can introduce also direct couplings of the ψ_L fermions to the field V^i_μ by generalizing the procedure of [31, 32]. For each ψ_L we can construct the following $SU(2)$ doublets

$$\chi^i_L = \Sigma^\dagger_i \Sigma^\dagger_{i-1} \cdots \Sigma^\dagger_1 \psi_L, \quad i = 1, \ldots, K. \tag{8}$$

These fields transform under eqs.(1) as

$$\chi^i_L \to U_i \Sigma^\dagger_i U^\dagger_{i-1} U_{i-1} \Sigma^\dagger_{i-1} U^\dagger_{i-2} \cdots U^\dagger_1 U_1 \Sigma^\dagger_1 L^\dagger \psi'_L = U_i \Sigma^\dagger_i \cdots \Sigma^\dagger_1 L^\dagger L \psi_L = U_i \chi^i_L. \tag{9}$$

Therefore we can add to the fermion lagrangian in eq.(7) the following term containing direct left-handed fermion couplings to V^i_μ which is invariant under the symmetry transformation of the model:

$$\sum_{i=1}^K b_i \bar{\chi}^i_L i\gamma^\mu(\partial_\mu + ig_i V^i_\mu + \frac{i}{2}\tilde{g}'(B-L)\tilde{\mathcal{Y}}_\mu)\chi^i_L \tag{10}$$

where b_i are K dimensionless parameters. In the unitary gauge $\Sigma_i = I$ and therefore the total fermion lagrangian is given by

$$
\begin{aligned}
\mathcal{L}^{tot}_{fermions} &= \bar{\psi}_L i\gamma^\mu (\partial_\mu + i\tilde{g}\tilde{W}_\mu + \frac{i}{2}\tilde{g}'(B-L)\tilde{\mathcal{Y}}_\mu)\psi_L \\
&+ \sum_{i=1}^K b_i \bar{\psi}_L i\gamma^\mu (\partial_\mu + ig_i V^i_\mu + \frac{i}{2}\tilde{g}'(B-L)\tilde{\mathcal{Y}}_\mu)\psi_L \\
&+ \bar{\psi}_R i\gamma^\mu (\partial_\mu + i\tilde{g}'\tilde{Y}_\mu + \frac{i}{2}\tilde{g}'(B-L)\tilde{\mathcal{Y}}_\mu)\psi_R \,.
\end{aligned}
\tag{11}
$$

The canonical kinetic term for fermions is obtained by the following redefinition

$$
\psi_L \rightarrow \frac{1}{\sqrt{1 + \sum_{i=1}^K b_i}}\psi_L \,,
\tag{12}
$$

so that the final fermion coupling lagrangian is given by

$$
\begin{aligned}
\mathcal{L}^{tot}_{fermions} &= \bar{\psi}_L i\gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R i\gamma^\mu \partial_\mu \psi_R \\
&+ \frac{1}{1 + \sum_{i=1}^K b_i}\bar{\psi}_L i\gamma^\mu (i\tilde{g}\tilde{W}_\mu + \frac{i}{2}\tilde{g}'(B-L)\tilde{\mathcal{Y}}_\mu)\psi_L \\
&+ \sum_{i=1}^K \frac{b_i}{1 + \sum_{j=1}^K b_j}\bar{\psi}_L i\gamma^\mu (ig_i V^i_\mu + \frac{i}{2}\tilde{g}'(B-L)\tilde{\mathcal{Y}}_\mu)\psi_L \\
&+ \bar{\psi}_R i\gamma^\mu (i\tilde{g}'\tilde{Y}_\mu + \frac{i}{2}\tilde{g}'(B-L)\tilde{\mathcal{Y}}_\mu)\psi_R \,.
\end{aligned}
\tag{13}
$$

IV. THE LOW-ENERGY LIMIT

Let us study the effects of the V_i $(i = 1,\ldots,K)$ particles in the low-energy limit. This can be done by eliminating the V_i fields with the solution of their equations of motion for $g_i \gg 1$, limit that corresponds to heavy masses for the V_i fields (see eq.(6)). In fact in this limit the kinetic term of the new resonances is negligible. The corresponding effective theory will be considered up to order $(1/g_i)^2$.

Let us solve the equations of motion for the field V_i in terms of $\tilde{W}$ and $\tilde{Y}$ (see Appendix A). For the moment being we neglect fermion current contributions which give current-current interactions in the effective lagrangian, these will be considered later on. By separating charged and neutral components, we get

$$
V^\alpha_i = \frac{1}{g_i}(\tilde{g}\tilde{W}^\alpha z_i)\,, \quad \alpha = 1,2\,,
\tag{14}
$$

$$
V^3_i = \frac{1}{g_i}(\tilde{g}'\tilde{\mathcal{Y}}_\mu y_i + \tilde{g}\tilde{W}^3 z_i)\,,
\tag{15}
$$

where we have, for convenience, introduced the following variables

$$z_i = \sum_{j=i+1}^{K+1} x_j, \quad x_i = \frac{f^2}{f_i^2}, \quad \frac{1}{f^2} = \sum_{i=1}^{K+1} \frac{1}{f_i^2}, \quad \sum_{i=1}^{K+1} x_i = 1, \quad y_i = 1 - z_i. \tag{16}$$

By using the standard linear combinations

$$\tilde{A}_\mu = s_{\tilde{\theta}} \tilde{W}_\mu^3 + c_{\tilde{\theta}} \tilde{\mathcal{Y}}_\mu,$$
$$\tilde{Z}_\mu = c_{\tilde{\theta}} \tilde{W}_\mu^3 - s_{\tilde{\theta}} \tilde{\mathcal{Y}}_\mu, \tag{17}$$

with $s_{\tilde{\theta}}$ and $c_{\tilde{\theta}}$ defined as in the SM,

$$\tilde{e} = \tilde{g} s_{\tilde{\theta}} = \tilde{g}' c_{\tilde{\theta}} \tag{18}$$

and by substituting in the quadratic part of the kinetic lagrangian, we obtain

$$\mathcal{L}_{eff}^{kin\ (2)}(\tilde{W}^\pm, \tilde{A}, \tilde{Z}) = -\frac{1}{4}(1 + z_\gamma)\tilde{A}_{\mu\nu}\tilde{A}^{\mu\nu} - \frac{1}{2}(1 + z_w)\tilde{W}_{\mu\nu}^+\tilde{W}^{\mu\nu-}$$
$$- \frac{1}{4}(1 + z_z)\tilde{Z}_{\mu\nu}\tilde{Z}^{\mu\nu} + \frac{1}{2}z_{z\gamma}\tilde{A}_{\mu\nu}\tilde{Z}^{\mu\nu}, \tag{19}$$

where $O_{\mu\nu} = \partial_\mu O_\nu - \partial_\nu O_\mu$, $(O = \tilde{W}^\pm, \tilde{A}, \tilde{Z})$ and

$$z_\gamma = \sum_{i=1}^K \left(\frac{\tilde{e}}{g_i}\right)^2, \quad z_w = \sum_{i=1}^K \left(\frac{\tilde{g}}{g_i}\right)^2 z_i^2, \quad z_z = \frac{\tilde{e}^2}{s_{\tilde{\theta}}^2 c_{\tilde{\theta}}^2}\sum_{i=1}^K \frac{1}{g_i^2}\left(z_i - s_{\tilde{\theta}}^2\right)^2, \quad z_{z\gamma} = -\frac{\tilde{e}^2}{s_{\tilde{\theta}}c_{\tilde{\theta}}}\sum_{i=1}^K \frac{1}{g_i^2}\left(z_i - s_{\tilde{\theta}}^2\right). \tag{20}$$

By making use of the solutions of the equations of motion for V_i in the fermion lagrangian, we obtain

$$\mathcal{L}_{eff}^{charged} = -\frac{\tilde{e}}{\sqrt{2}s_{\tilde{\theta}}}(1 - \frac{b}{2})\overline{\psi}_d\gamma^\mu\frac{1 - \gamma_5}{2}\psi_u\tilde{W}_\mu^- + h.c., \tag{21}$$

$$\mathcal{L}_{eff}^{neutral} = -\frac{\tilde{e}}{s_{\tilde{\theta}}c_{\tilde{\theta}}}(1 - \frac{b}{2})\overline{\psi}\gamma^\mu\left[T_L^3\frac{1 - \gamma_5}{2} - \frac{Qs_{\tilde{\theta}}^2}{(1 - \frac{b}{2})}\right]\psi\tilde{Z}_\mu - \tilde{e}\overline{\psi}\gamma^\mu Q\psi\tilde{A}_\mu, \tag{22}$$

with

$$b = 2\frac{\sum_{i=1}^K b_i y_i}{1 + \sum_{i=1}^K b_i} \tag{23}$$

and $T_L^3\psi_L = \tau_3/2\psi_L$, $T_L^3\psi_R = 0$.

As shown in Appendix B, the additional fermion direct couplings to V_i give rise to the following current-current interaction term:

$$\mathcal{L}_{eff}^{quart} = \beta\sum_{a=1}^3 \left(\overline{\psi}_L\gamma^\mu\frac{\tau^a}{2}\psi_L\right)^2 \tag{24}$$

with

$$\beta = \frac{1}{8f^2}\left(\bar{b}_K - b\right)^2 - \frac{1}{8f^2}\sum_{i=1}^{K} x_{i+1}\bar{b}_i^2 \tag{25}$$

and

$$\bar{b}_i = 2\frac{\sum_{j=1}^{i} b_j}{1 + \sum_{j=1}^{K} b_j} \quad (i = 1, \cdots, K). \tag{26}$$

V. FIELDS AND COUPLINGS RENORMALIZATION

The corrections to the quadratic part of the kinetic lagrangian given in eq.(19) are $U(1)_{em}$ invariant and produce a wave-function renormalization of $\tilde{A}_\mu, \tilde{Z}_\mu, \tilde{W}_\mu^\pm$ plus a mixing term $\tilde{A}_\mu - \tilde{Z}_\mu$. Notice that in general there could be two other renormalization terms: $\delta M_W^2 \tilde{W}_\mu^+ \tilde{W}^{\mu-}$ and $\delta M_Z^2 \tilde{Z}_\mu \tilde{Z}^\mu$ which, however, are zero in this model. To identify the physical quantities we define new fields in such a way to have canonical kinetic terms and to cancel the mixing term $\tilde{A}_\mu - \tilde{Z}_\mu$. They are given by the following relations:

$$\begin{aligned}
\tilde{A}_\mu &= (1 - \frac{z_\gamma}{2})A_\mu + z_{z\gamma}Z_\mu\,, \\
\tilde{W}_\mu^\pm &= (1 - \frac{z_w}{2})W_\mu^\pm\,, \\
\tilde{Z}_\mu &= (1 - \frac{z_z}{2})Z_\mu\,.
\end{aligned} \tag{27}$$

Let us study the effects of this renormalization. First of all for the mass terms we get:

$$- f^2 \text{tr}(\tilde{W}_\mu - \tilde{Y}_\mu)^2 = -\tilde{M}_W^2(1 - z_w)W^{\mu+}W_\mu^- - \frac{1}{2}\tilde{M}_Z^2(1 - z_z)Z^\mu Z_\mu \tag{28}$$

where, for comparison with the SM results, we have defined $f = v/2$ and

$$\tilde{M}_W^2 = \frac{v^2}{4}\tilde{g}^2\,, \qquad \tilde{M}_Z^2 = \tilde{M}_W^2/c_{\tilde{\theta}}^2\,. \tag{29}$$

Also, the field renormalization affects all the couplings of the standard gauge bosons to the fermions. By substituting eq.(27) in eqs.(21) and (22) we get

$$\mathcal{L}_{eff}^{charged} = -\frac{\tilde{e}}{\sqrt{2}s_{\tilde{\theta}}}(1 - \frac{b}{2})(1 - \frac{z_w}{2})\overline{\psi}_d\gamma^\mu\frac{1-\gamma_5}{2}\psi_u W_\mu^- + h.c.\,, \tag{30}$$

$$\begin{aligned}
\mathcal{L}_{eff}^{neutral} &= -\frac{\tilde{e}}{s_{\tilde{\theta}}c_{\tilde{\theta}}}(1 - \frac{b}{2})(1 - \frac{z_z}{2})\overline{\psi}\gamma^\mu\Big[T_L^3\frac{1-\gamma_5}{2} - Qs_{\tilde{\theta}}^2\frac{1 - \frac{c_{\tilde{\theta}}}{s_{\tilde{\theta}}}z_{z\gamma}}{1 - \frac{b}{2}}\Big]\psi Z_\mu \\
&\quad -\tilde{e}(1 - \frac{z_\gamma}{2})\overline{\psi}\gamma^\mu Q\psi A_\mu\,.
\end{aligned} \tag{31}$$

We see that the physical constants as the electric charge, the Fermi constant and the mass of the Z must be redefined in terms of the parameters appearing in our effective lagrangian. They are identified as follows

$$
\begin{aligned}
e &= \tilde{e}\left(1 - \frac{z_\gamma}{2}\right), \\
M_Z^2 &= \tilde{M}_Z^2(1 - z_z).
\end{aligned}
\tag{32}
$$

Concerning the Fermi constant G_F, it is evaluated from the μ-decay process. Taking into account the modified charged current coupling, the W mass

$$
M_W^2 = \tilde{M}_W^2(1 - z_w)
\tag{33}
$$

and the charged current-current interaction we obtain

$$
\frac{G_F}{\sqrt{2}} = \frac{1}{8}\tilde{g}^2\left(1 - \frac{b}{2}\right)^2 \frac{1 - z_w}{M_W^2} + \frac{1}{4}\beta
\tag{34}
$$

where we have used eqs. (30) and (24).

Following [37] we define s_θ by

$$
\frac{G_F}{\sqrt{2}} = \frac{e^2}{8s_\theta^2 c_\theta^2 M_Z^2}.
\tag{35}
$$

By comparing with eq.(34) we get

$$
s_{2\tilde{\theta}} = s_{2\theta}\sqrt{X}
\tag{36}
$$

where

$$
X = \frac{\left(1 - \frac{b}{2}\right)^2}{1 - \frac{\sqrt{2}\beta}{4G_F}}(1 + z_\gamma - z_z).
\tag{37}
$$

More explicitly

$$
s_{\tilde{\theta}}^2 = \frac{1}{2} - \frac{1}{2}\sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_F M_Z^2}X}
\tag{38}
$$

with α the fine structure constant. Notice that for $X = 1$ we recover the standard definition for s_θ.

VI.   THE ϵ PARAMETERS

The corrections to the tree-level SM results are usually parameterized in terms of a set of parameters called $\epsilon_{1,2,3}$ that can be obtained from Δr_W, $\Delta\rho$ and Δk [37, 38]. Let us start

from Δr_W defined by:

$$\frac{M_W^2}{M_Z^2} = c_\theta^2 \left[1 - \frac{s_\theta^2}{c_{2\theta}} \Delta r_W \right] . \tag{39}$$

From the relation $\tilde{M}_W^2 = \tilde{M}_Z^2 c_{\tilde{\theta}}^2$, and using eq.(38), we get

$$\frac{M_W^2}{M_Z^2} = c_\theta^2 \left[(1 + z_z - z_w) \left(\frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2} G_F M_Z^2} X} \right) \right] \tag{40}$$

so, for comparison,

$$\begin{aligned}
\Delta r_W &= \frac{c_{2\theta}}{s_\theta^2} \left[1 - \frac{c_{\tilde{\theta}}^2}{c_\theta^2} (1 + z_z - z_w) \right] \\
&= \frac{c_{2\theta}}{s_\theta^2 c_\theta^2} \left[c_\theta^2 - \left(\frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2} G_F M_Z^2} X} \right) (1 + z_z - z_w) \right]
\end{aligned} \tag{41}$$

The definitions of $\Delta\rho$ and Δk are given in terms of the neutral current couplings to the Z gauge boson

$$\mathcal{L}^{neutral}(Z) = -\frac{e}{s_\theta c_\theta} \left(1 + \frac{\Delta\rho}{2} \right) Z_\mu \overline{\psi} [\gamma^\mu g_V + \gamma^\mu \gamma_5 g_A] \psi \tag{42}$$

with

$$\begin{aligned}
g_V &= \frac{T_L^3}{2} - s_\theta^2 Q , \\
g_A &= -\frac{T_L^3}{2} , \\
s_\theta^2 &= (1 + \Delta k) s_\theta^2 .
\end{aligned} \tag{43}$$

For comparison with eq.(31) we obtain

$$\begin{aligned}
\Delta\rho &= 2 \left[\frac{s_\theta c_\theta}{s_{\tilde{\theta}} c_{\tilde{\theta}}} \left(1 - \frac{b}{2} \right) \left(1 - \frac{z_z}{2} \right) \left(1 - \frac{z_\gamma}{2} \right)^{-1} - 1 \right] = 2 \left(\sqrt{1 - \frac{\beta}{2\sqrt{2}} \frac{1}{G_F}} - 1 \right) , \\
\Delta k &= -1 + \frac{s_{\tilde{\theta}}^2}{s_\theta^2} \frac{1}{\left(1 - \frac{b}{2} \right)} \left(1 - \frac{c_{\tilde{\theta}}}{s_{\tilde{\theta}}} z_{z\gamma} \right) ,
\end{aligned} \tag{44}$$

with $s_{\tilde{\theta}}$ given in eq.(38). Finally we can compute the new physics contribution to the ϵ parameters [37]

$$\begin{aligned}
\epsilon_1^N &= \Delta\rho , \\
\epsilon_2^N &= c_\theta^2 \Delta\rho + \frac{s_\theta^2}{c_{2\theta}} \Delta r_W - 2 s_\theta^2 \Delta k , \\
\epsilon_3^N &= c_\theta^2 \Delta\rho + c_{2\theta} \Delta k .
\end{aligned} \tag{45}$$

Expanding up to the first order in b_i and neglecting terms $O(b_i/g_i^2))$, we get

$$\epsilon_1^N \simeq 0\,,$$
$$\epsilon_2^N \simeq 0\,,$$
$$\epsilon_3^N \simeq \sum_{i=1}^{K} y_i\left(\frac{e^2}{s_\theta^2 g_i^2} z_i - b_i\right). \qquad (46)$$

This final expression suggests that the introduction of the b_i direct fermion couplings to V_i can compensate for the contribution of the tower of gauge vectors to ϵ_3. This would reconcile the Higgsless model with the electroweak precision measurements by fine-tuning the direct fermion couplings.

VII. PARTICULAR MODELS

In this section we consider the bounds on the parameter space of the linear moose model in some examples, by comparing the theoretical predicted values for the ϵ parameters with the experimental values [9]

$$\epsilon_1 = (5 \pm 1.1) \times 10^{-3}\,,$$
$$\epsilon_2 = (-8.8 \pm 1.2) \times 10^{-3}\,,$$
$$\epsilon_3 = (4.8 \pm 1.0) \times 10^{-3}\,. \qquad (47)$$

We assume the same radiative corrections for the ϵ parameters as in the SM with a cutoff $m_H = 1$ TeV and $m_{top} = 178$ GeV. We used the exact formulas in b_i given in eqs.(45), (41) and (44).

We first study the simplest model with all $f_i = const = f_c$, $g_i = const = g_c$ and $b_i = const = b_c$.

For $K = 1$ and $K = 10$ gauge groups we get the bounds from ϵ_2 and ϵ_3 shown in Fig. 2. The experimental value for ϵ_1 gives a 95% CL bound independent on g_c, $b_c \lesssim 0.14$ for $K = 1$ and $b_c \lesssim 0.025$ for $K = 10$.

The most stringent bound comes from ϵ_3, and it shows that a region exists where it is possible to satisfy the electroweak constraints for small values of Kb_c. As shown in the Fig. 2, the limits in these variables do not strongly depend on K because of a scaling property. It is obvious that for increasing K the allowed region in the parameter space $(b_c, 1/g_c)$ shrinks.
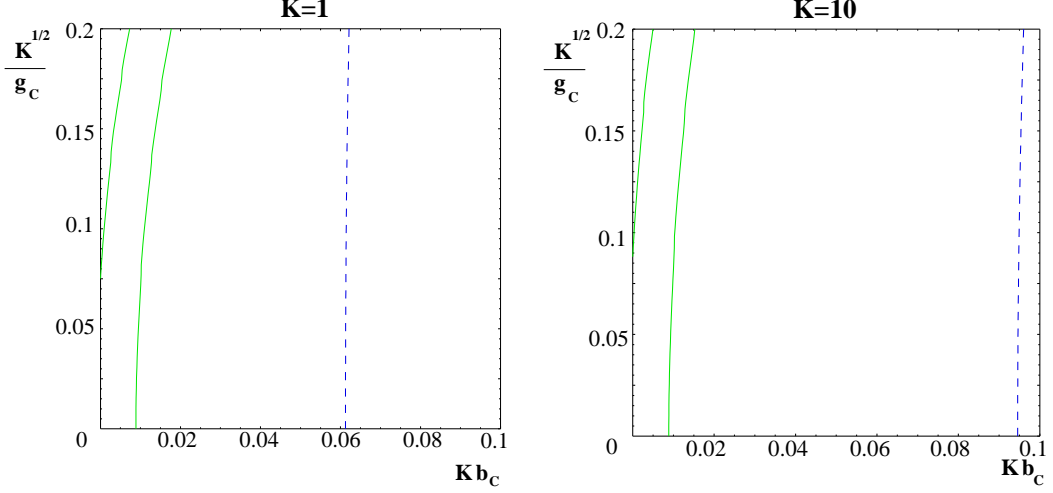
FIG. 2: *95% CL bounds on the parameter space $(Kb_c, \sqrt{K}/g_c)$ from the experimental values of ϵ_2 and ϵ_3 for $K = 1$ (left) $K = 10$ (right), assuming the SM radiative corrections for $m_H = 1$ TeV and $m_{top} = 178$ GeV. The allowed parameter space from ϵ_2 is the region to the left of the dashed line and from ϵ_3 the region between the continuous lines.*
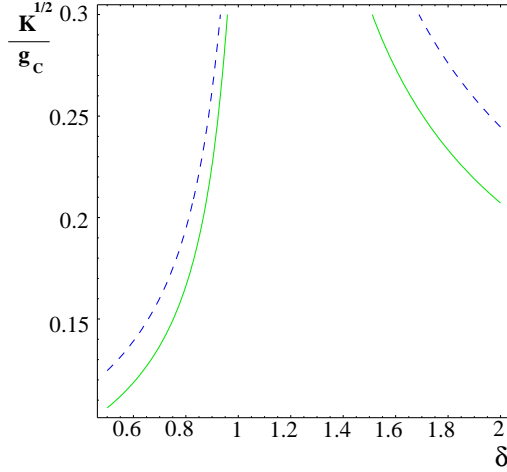

FIG. 3: *95% CL bounds on the parameter space $(\delta, \sqrt{K}/g_c)$ from the experimental value of ϵ_3 for $K = 1$ (continuous line), $K = 10$ (dash line), assuming the radiative corrections for $m_H = 1$ TeV and $m_{top} = 178$ GeV. The allowed parameter space is the region between the corresponding lines.*

The approximate expression for ϵ_3 given in eq.(46) suggests also the following choice for b_i,

$$b_i = \delta \frac{e^2}{s_\theta^2 g_i^2} z_i = \delta \frac{e^2}{s_\theta^2 g_i^2}(1 - y_i) \tag{48}$$

which gives $\epsilon_3^N \simeq 0$ for $\delta = 1$ for small b_i. Assuming again $f_i = f_c$, $g_i = g_c$, this means

$$b_i = \delta \frac{e^2}{s_\theta^2 g_c^2} \left(1 - \frac{i}{K+1} \right) . \tag{49}$$

In Fig. 3 we plot the 95% CL bounds on the parameter space $(\delta, \sqrt{K}/g_c)$ from the experimental value of ϵ_3 for $K = 1$ (continuous line), $K = 10$ (dash line), assuming the radiative corrections for $m_H = 1$ TeV and $m_{top} = 178$ GeV. For $K \gg 10$ the allowed region is nearly independent on K. Very loose bounds are obtained from ϵ_1 and ϵ_2.

In conclusion, by fine tuning every direct fermion coupling in each site in a way to compensate the corresponding contribution to ϵ_3 from the linear moose, a sizeable region in the parameter space is left.

VIII.   CONTINUUM LIMIT

It is known that the discretization of a gauge theory lagrangian in a 4+1 dimensional space-time along the fifth dimension (the segment of length πR) gives rise to a linear moose chiral lagrangian after a suitable identification of the gauge and link couplings [10, 11, 12, 13, 39]. We consider the continuum limit $a \to 0$, $K \to \infty$ with the condition $Ka = \pi R$, where πR is the length of the segment in the fifth dimension. We would like to discuss what is the continuum limit for the direct fermionic couplings when we choose the b_i's according to the eq.(48) with $\delta = 1$ for simplicity. By defining

$$\lim_{a \to 0} \frac{b_i}{a} = b(y), \quad \lim_{a \to 0} a f_i^2 = f^2(y), \quad \lim_{a \to 0} a g_i^2 = g_5^2(y) \tag{50}$$

we find, assuming $g_5(y) = g_5$ with g_5 a constant

$$b(y) = \frac{e^2}{s_\theta^2 g_5^2} \int_y^{\pi R} dt \frac{f^2}{f^2(t)} \tag{51}$$

with

$$\frac{1}{f^2} = \int_0^{\pi R} \frac{dy}{f^2(y)} . \tag{52}$$

From eq. (51) we see that

$$b(0) = \frac{e^2}{s_\theta^2 g_5^2}, \quad b(\pi R) = 0 . \tag{53}$$

Therefore the direct fermionic coupling decreases along the fifth dimension going from the brane located at $y = 0$ to the brane at $y = \pi R$. For the case of constant $f(y) = \bar{f}$ we find

$$b(y) = \frac{e^2}{s_\theta^2 g_5^2} \left(1 - \frac{y}{\pi R} \right) . \tag{54}$$

With this choice the contribution from the new delocalized fermion interactions to ϵ_3^N is given by

$$\epsilon_3^N|_{ferm} = -\frac{1}{\pi R} \int_0^{\pi R} dy \; y b(y) = -\frac{e^2}{s_\theta^2 g_5^2} \frac{\pi R}{6} \tag{55}$$

which is just the opposite of the contribution to ϵ_3^N in the linear moose [15, 17].

Another interesting case corresponds to a Randall-Sundrum metric along the fifth dimension [39, 40], that is

$$f(y) = \bar{f} e^{ky} \tag{56}$$

and we find

$$b(y) = \frac{e^2}{s_\theta^2 g_5^2} \frac{e^{-2\pi kR} - e^{-2ky}}{e^{-2\pi kR} - 1} . \tag{57}$$

In this case we get

$$\epsilon_3^N|_{ferm} = -\int_0^{\pi R} dy \frac{e^{-2ky} - 1}{e^{-2k\pi R} - 1} b(y) = -\frac{e^2}{s_\theta^2 g_5^2} \frac{1}{4k} \frac{e^{4k\pi R} - 4k\pi R e^{2k\pi R} - 1}{(1 - e^{2k\pi R})^2} \tag{58}$$

which is the opposite of the contribution from the gauge bosons derived in [17].

Summarizing, in the continuum limit, left- and right-handed fermions live at the opposite ends of the extra-dimension; they feel only the SM gauge transformations. However, in the discrete, we have introduced an interaction term invariant under all the symmetries of the model which delocalizes the left-handed fermions in the continuum limit. In fact, we have seen in eq.(8) that the fermionic fields along the string are defined in terms of the operator

$$\Sigma_1 \Sigma_2 \cdots \Sigma_i . \tag{59}$$

In five-dimensions the fields Σ's can be interpreted as the link variables along the fifth dimension. As such they can be written in terms the fifth component of the heavy gauge fields V. As a consequence the operator given in eq.(59) becomes a Wilson line

$$\Sigma_1 \Sigma_2 \cdots \Sigma_i \rightarrow P\left(\exp\left(i \int_0^y dt V_5(t, x)\right)\right) . \tag{60}$$

In this way the original fermionic fields acquire a non-local interaction induced by Wilson lines.

This non-local interaction is the origin of the possible negative contribution to the parameter ϵ_3.

IX. CONCLUSIONS

Models with replicas of gauge groups have been recently considered because they appear in the deconstruction of five dimensional gauge models which have been used to describe the electroweak breaking without the Higgs [10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21]. The four dimensional description is based on the linear moose lagrangians that were already proposed in technicolor and composite Higgs models [41]. In general these models satisfy the constraints arising from the parameters T and U (or ϵ_1 and ϵ_2) due to the presence of a custodial $SU(2)$ symmetry. However they generally give a correction of order $\mathcal{O}(1)$ ($\mathcal{O}(10^{-2})$) to the parameter S (ϵ_3). In this paper we have considered a linear moose based on replicas of $SU(2)$ gauge groups, with the electroweak gauge groups $SU(2)_L$ and $U(1)_Y$ and ordinary fermions at the two ends of the moose string. Within this framework it seems that the only way to satisfy the electroweak constraints is the one considered in [16, 17] which, however, sets the unitarity violation scale around 1 TeV, as in the SM without a Higgs. In this paper we have considered the possibility of raising up this scale by allowing the fermions to interact with the gauge fields along the string. This is realized through the introduction of a string of scalar fields which, in the continuum limit, is equivalent to delocalize the left-handed fermions on the boundary with a Wilson line. This new non local interaction gives a contribution to ϵ_3 of opposite sign with respect to the one coming from the gauge fields along the string. Therefore, at the expenses of some fine tuning, it is possible to satisfy the experimental limits. At the same time the scale of the heavy vector bosons can be lowered allowing a corresponding increasing of the unitarity bound.

After this work was completed, a related paper appeared [42] where a partial delocalization of fermions was considered.

APPENDIX A

The covariant derivatives of eq.(3) can be expressed in a compact form, by defining $V_\mu^0 = \tilde{W}_\mu$, $V_\mu^{K+1} = \tilde{Y}_\mu$, $\tilde{g} = g_0$, $\tilde{g}' = g_{K+1}$,

$$D_\mu \Sigma_i = \partial_\mu \Sigma_i - i g_{i-1} V_\mu^{i-1} \Sigma_i + i \Sigma_i g_i V_\mu^i, \quad i = 1, \cdots, K+1. \tag{A1}$$

The lagrangian for the mass terms of the gauge fields and their fermion couplings can be rewritten

$$\mathcal{L}_{mass} = \frac{1}{2} \sum_{i=1}^{K+1} f_i^2 B_{i-1}^2 - \sum_{i=0}^{K+1} J_i^a V_i^a. \tag{A2}$$

The variables B_i are the analogue in the discrete formulation of canonical momenta and are given by

$$B_i = g_i V_i - g_{i+1} V_{i+1}, \quad i = 0, \cdots, K, \tag{A3}$$

the $V_i^a = Tr(V_i \tau^a)$ and J_i^a are the related fermionic currents. For $i = 0, K+1$ J_i are the SM fermionic currents, while

$$J_i^a = -g_i \frac{b_i}{1 + \sum_{j=1}^{K} b_j} \bar{\psi}_L \gamma^\mu \frac{\tau^a}{2} \psi_L, \quad i = 1, \cdots, K. \tag{A4}$$

Notice that the $K+1$ fields B_i are not independent, since

$$\sum_{i=0}^{K} B_i = g_0 V_0 - g_{K+1} V_{K+1}. \tag{A5}$$

Therefore we solve the equations of motion, which involve three nearest neighborhoods, by solving first in the B_i's and then inverting the relation between the B_i's and the fields V_i. These equations involve only first neighborhoods. This is the analogue of converting a second order differential equation in a pair of first order equations.

The equations of motion can be written in the following form

$$- f_i^2 B_{i-1} + f_{i+1}^2 B_i = L_i, \quad i = 1, \cdots, K, \tag{A6}$$

where we have redefined the sources as

$$L_i = \frac{J_i}{g_i}. \tag{A7}$$

We can solve for all the B_i, $i = 1, \cdots, K$ in terms of B_0 finding

$$B_i = \frac{1}{f_{i+1}^2} \left(\sum_{j=1}^{i} L_j + f_1^2 B_0 \right), \quad i = 1, \cdots, K. \tag{A8}$$

It is convenient to introduce the following variables

$$\frac{1}{f^2} = \sum_{i=1}^{K+1} \frac{1}{f_i^2}, \quad x_i = \frac{f^2}{f_i^2}, \quad i = 1, \cdots, K+1, \tag{A9}$$

$$y_i = \sum_{j=1}^{i} x_j, \quad z_i = \sum_{j=i+1}^{K+1} x_j, \tag{A10}$$

with the properties

$$y_i + z_i = 1, \quad y_1 = x_1, \quad z_K = x_{K+1}. \tag{A11}$$

By summing the eqs.(A8) over i from 1 to K and using eq.(A5) we get a relation for B_0 which can be easily solved obtaining

$$B_0 = -\frac{x_1}{f^2} \sum_{i=1}^{K} z_i \, L_i + x_1(g_0 V_0 - g_{K+1} V_{K+1}), \tag{A12}$$

and

$$B_i = \frac{x_{i+1}}{f^2} \left(\sum_{j=1}^{i} y_j \, L_j - \sum_{j=i+1}^{K} z_j \, L_j + f^2(g_0 V_0 - g_{K+1} V_{K+1}) \right). \tag{A13}$$

By using the discrete step function given by

$$\theta_{i,j} = \begin{cases} 1, & \text{for } i \geq j \\ 0, & \text{for } i < j \end{cases} \tag{A14}$$

we can write

$$B_i = \frac{x_{i+1}}{f^2} \sum_{j=1}^{K} (\theta_{i,j} y_j - \theta_{j,i+1} z_j) \, L_j + f^2(g_0 V_0 - g_{K+1} V_{K+1}), \quad i = 0, \cdots, K. \tag{A15}$$

Further we need to reexpress the fields V_i in terms of the B_i's. We find

$$\begin{aligned} V_i &= \frac{1}{g_i} g_{K+1} V_{K+1} + \frac{1}{g_i} \sum_{j=1}^{K} \theta_{j,i} B_j \\ &= \frac{1}{g_i} \left[g_0 V_0 z_i + g_{K+1} V_{K+1} y_i + \sum_{j=1}^{K} \theta_{j,i} x_{i+1} \sum_{l=1}^{K} (\theta_{j,l} y_l - \theta_{l,j+1} z_l) \, L_l \right]. \end{aligned} \tag{A16}$$

If we neglect fermion currents and we separate charged and neutral components we obtain

$$V_i^{\alpha} = \frac{1}{g_i} (\tilde{g} \tilde{W}^{\alpha} z_i), \tag{A17}$$

$$V_i^3 = \frac{1}{g_i} (\tilde{g}' \tilde{\mathcal{Y}} y_i + \tilde{g} \tilde{W}^3 z_i). \tag{A18}$$

APPENDIX B

In this appendix we evaluate the quartic terms in the fermion fields arising after having eliminated the heavy fields V_i, $i = 1, \cdots, K$. Let us start again from the initial lagrangian excluding the kinetic terms as given in eq.(A2)

$$\mathcal{L}_{mass} = \frac{1}{2} \sum_{i=1}^{K+1} f_i^2 B_{i-1}^2 - \sum_{i=1}^{K} J_i^a V_i^a - J_0 V_0 - J_{K+1} V_{K+1}. \tag{B1}$$

The solutions in eq.(A15) for the fields B_i can be expressed as

$$B_i = \bar{B}_i + x_{i+1}C, \quad C = (g_0 V_0 - g_{K+1} V_{K+1}), \quad i = 0, \cdots, K. \tag{B2}$$

where $\bar{B}_i$ are the solutions when the fields V_0 and V_{K+1} (the standard fields $\tilde{W}$ and $\tilde{Y}$) are turned off, that is

$$\bar{B}_0 = -\frac{x_1}{f^2} \sum_{i=1}^{K} z_i L_i, \quad \bar{B}_i = \frac{x_{i+1}}{f^2} \sum_{j=1}^{K} (\theta_{i,j} - z_j) L_j. \tag{B3}$$

Let us notice that the terms in J_0, J_{K+1} and the ones given by the currents J_i times the standard field combination C, contribute to the total fermionic current already evaluated in the text and given in eq.(13). Therefore we can subtract them and we are left with

$$\mathcal{L}' = \frac{1}{2} \sum_{i=1}^{K+1} f_i^2 \left(\bar{B}_{i-1} + x_i C \right)^2 - \sum_{i=1}^{K} J_i \bar{V}_i, \tag{B4}$$

where the fields $\bar{V}_i$ are the fields V_i with the standard model contribution subtracted, that is, using eq.(A16)

$$\bar{V}_i = \frac{1}{g_i} \sum_{j=1}^{K} \theta_{j,i} \bar{B}_j. \tag{B5}$$

We see immediately that the terms linear in C vanishes. In fact

$$\sum_{i=1}^{K+1} f_i^2 x_i \bar{B}_{i-1} C = f^2 \left(\sum_{i=1}^{K+1} \bar{B}_{i-1} \right) C = 0 \tag{B6}$$

due to the identity satisfied by the fields $\bar{B}_i$ (see eq.(A5)). On the other hand the term in C^2 gives rise to the $\tilde{W}$ and $\tilde{Z}$ masses since

$$\frac{1}{2} \sum_{i=1}^{K+1} f_i^2 x_i^2 C^2 = \frac{1}{2} f^2 (\tilde{g} \tilde{W} - \tilde{g}' \tilde{Y})^2 \tag{B7}$$

from which

$$\tilde{M}_W^2 = f^2 \tilde{g}^2, \quad \tilde{M}_Z^2 = \frac{\tilde{M}_W^2}{c_\theta^2}. \tag{B8}$$

Therefore the quartic term is obtained from

$$\mathcal{L}_{eff}^{quart} = \frac{1}{2} \sum_{i=1}^{K+1} f_i^2 \bar{B}_{i-1}^2 - \sum_{i=1}^{K} J_i \bar{V}_i. \tag{B9}$$

After substitution we get

$$\mathcal{L}_{eff}^{quart} = \frac{1}{2f^2} \sum_{j,\ell=1}^{K} z_j z_\ell L_j L_\ell - \frac{1}{2} \sum_{i,j,\ell=1}^{K} \frac{1}{f_{i+1}^2} \theta_{i,j} \theta_{i,\ell} L_j L_\ell, \tag{B10}$$

or

$$\mathcal{L}_{eff}^{quart} = \frac{1}{2f^2}\left(\sum_{i=1}^{K} z_i L_i\right)^2 - \frac{1}{2f^2}\sum_{i=1}^{K} x_{i+1}\left(\sum_{j=1}^{i} L_j\right)^2 . \tag{B11}$$

Since

$$L_i = \frac{b_i}{1 + \sum_{i=1}^{K} b_i}\bar{\psi}_L\gamma^\mu\frac{\tau^a}{2}\psi_L \tag{B12}$$

we find

$$\mathcal{L}_{eff}^{quart} = \beta\sum_a\left(\bar{\psi}_L\gamma^\mu\frac{\tau^a}{2}\psi_L\right)^2 \tag{B13}$$

with

$$\beta = \frac{1}{8f^2}\left(\bar{b}_K - b\right)^2 - \frac{1}{8f^2}\sum_{i=1}^{K} x_{i+1}\bar{b}_i^2 \tag{B14}$$

where

$$b = 2\frac{\sum_{i=1}^{K} y_i b_i}{1 + \sum_{i=1}^{K} b_i}, \quad \bar{b}_i = 2\frac{\sum_{j=1}^{i} b_j}{1 + \sum_{j=1}^{K} b_j} . \tag{B15}$$

As an example let us consider the simple case

$$b_i = b_c, \quad f_i = f_c . \tag{B16}$$

It follows

$$f^2 = \frac{f_c^2}{K+1}, \quad x_i = \frac{1}{K+1}, \quad y_i = \frac{i}{K+1} \tag{B17}$$

and

$$\bar{b}_i = 2\frac{ib_c}{1 + Kb_c}, \quad b = \frac{Kb_c}{1 + Kb_c} . \tag{B18}$$

Therefore

$$\beta = \frac{1}{8f^2}\frac{K^2 b_c^2}{(1 + Kb_c)^2} - \frac{1}{2f^2(K+1)}\frac{b_c^2}{(1 + Kb_c)^2}\sum_{i=1}^{K} i^2 = -\frac{1}{24f^2}\frac{(Kb_c)^2}{(1 + Kb_c)^2}\frac{K+2}{K} \tag{B19}$$

or, in terms of the parameter b,

$$\beta = -\frac{b^2}{24f^2}\frac{K+2}{K} . \tag{B20}$$

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